# Reinforcement of edge waves by beach cusps

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**Summary**. Beach cusps, formed during a storm are observed. The storm acted for three hours together with high water conditions. These beach cusps exhibited a quasi-uniform wavelength of 8 m. The measured topography after the storm, the calculated drift velocity of the incident wave and the synchronous edge wave are similar in scale and shape. The over one wavelength measured grain size is also correlated to the topography.
A nonlinear hydro-numerical model is used to investigate the reaction of edge waves on alongshore change in bottom topography. Edge waves are greatly amplified over beach cusps. The relative amplification of edge waves on beach cusps is more than 17 times that without a change in longshore topography. Amplification is slightly greater for edge waves travelling in the same direction as the longshore current during the storm.
An almost random positioning of sediment starts a feedback loop, which allows edge waves to grow and build short periodic cusps.

Keywords: beach cusps, edge waves Baltic Sea, German Coast, Zingst peninsula

## Introduction

The coastal profile can vary considerably over a year or even during a single storm event. Longshore bars can be formed and existing bars can be shifted or destroyed. The problem of sediment transport modelling is not yet completely solved - particularly the resulting direction of sediment transport outside the surf zone and the detailed morphology of the bars, which must be investigated more thoroughly. The wave theory with an onshore/offshore sediment-transport model suggests that the undertow gives an offshore-directed transport in the surf zone and that the transport outside the surf zone is weak, with a tendency to be in the onshore direction. This means that a longshore bar will tend to form on a constant-slope profile as a result of the cross-shore transport (Fredsøe and Deigaard, 1992). However these bars are forming offshore continuously and are not stable. Schönfeldt (1989, 1991) constructed an onshore/offshore sediment-transport model on the basis of edge waves. In this model, the bars move toward the beach. It seems that in nature both wave processes take part.
In principle, numerical models include edge waves, but excitation is only possible in non-linear models. On the other hand, the boundary conditions are simple only for standing edge waves. For effective excitation of edge waves, the alongshore extension of the model must be big enough that excitation and dissipation can reach equilibrium. On the cross-shore boundaries, these waves leave the model. An inject of these waves on the opposite boundary is possible only where there are identical depth profiles of at least 1.5 wave length on the cross-shore boundaries as well as other restrictions.

At beaches with bars, edge waves may be trapped at the location of the bar, their cross-shore shapes strongly amplified relative to their shoreline values (Kirby et al. 1981, Schönfeldt, 1991, Bryan and Bowen, 1996, 1998), with the strongest amplification occurring at incident-wave frequencies. The trapping of edge waves on near shore wave-guides, such as bars and longshore currents, has been presented as a mechanism for the growth, movement, or even formation of longshore bars (Schönfeldt, 1989, 1991, 1995; Howd et al.,1992; Bryan and Bowen, 1996, 1998).
Of particular interest have been shoreline features that show a regular longshore periodicity down the beach, variously described as beach cusps, sand waves, rhythmic topography, or grain cusps (Hom-ma and Sonu, 1963; Dolan et al., 1974; Guza and Inman, 1975; Komar, 1981). Beach cusps are crescentic shoreline features, concave seaward, that are characterised by a quasi-uniform longshore wavelength ranging from less than 3 m to at least 30 m (Dolan et al., 1974). Cusp formation is usually rapid, requiring only a few hours (Miller et al., 1989).
Holman and Bowen (1982) show how a variety of regular longshore patterns may be generated by two edge waves of the same frequency but different mode. Superimposition of incident waves on edge waves also generates longshore periodicity in drift velocity (Sallenger, 1979, Schwarzer, 1989).
There is a second explanation of beach cusps formation, the self-organisation theory using more advanced computer simulations (Werner and Fink, 1993, Coco et al., 1999). There is a positive feedback in areas with lower relief and a negative feedback will decrease the amount of net erosion and deposition within a well formed cusp. The feedback between the morphology of the beach and the flow of the water creates relief patterns. Almar et al. (2008) were unable to conclusively refute any of the mechanisms causing beach cusp formation since both existing theories, standing edge wave and self-organization, can predict the trend in the observed beach cusp spacing. We will show that in the presented case the beach cusp grow and spacing is a result of superposition of incident wave and synchronous edge wave and that the so formed topography will starts a feedback loop, which allows edge waves to grow and build short periodic cusps.

## Location of study

Measurements of wave characteristics, water level, and the topography after a storm were carried out in the western part of the Baltic Sea on the Zingst peninsula. On the measuring point there is a negligible tide of 0.1 m. The coast of the Zingst peninsula is a straight sandy coast, 18 km long and east-west orientated. The measuring point of the University of Leipzig is situated in the centre of this. The coast is protected by arrayed groins, approximately 100-150 m apart.

## Storm characteristics

On January 25th to 26th 1993 the mean wind velocity was for a relatively short time of 6 hours 17 m/s from NW. The storm caused for a short time a water-level rise of 1.17 m above mean water level (see Figure 1). During this time, the waves increased up to a significant wave height of $H_S$ =1.5 m measured 100 m from the coastline at a depth of

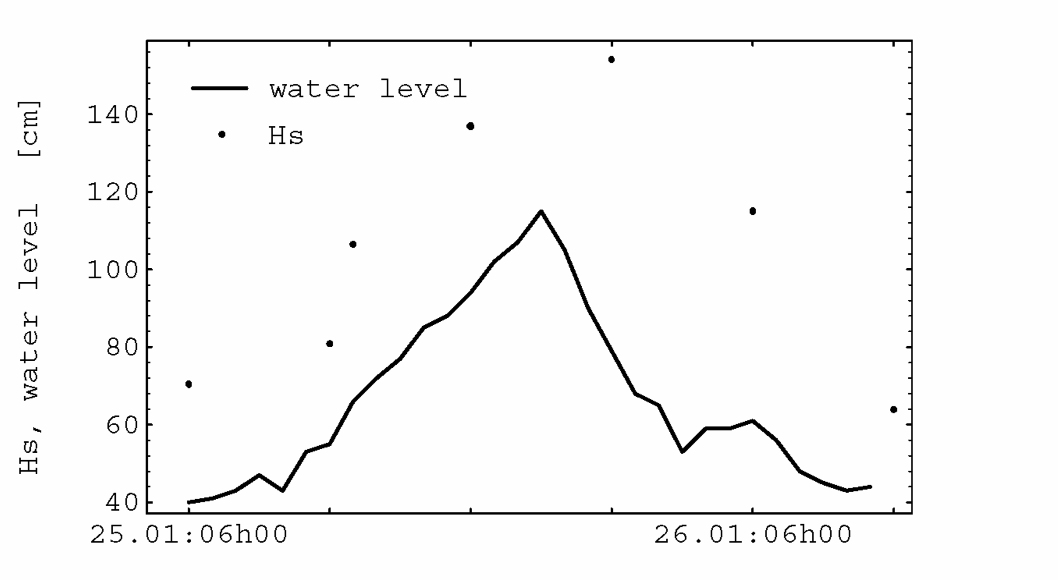


Fig. 1 Time series of water level and significant wave height $H_s$

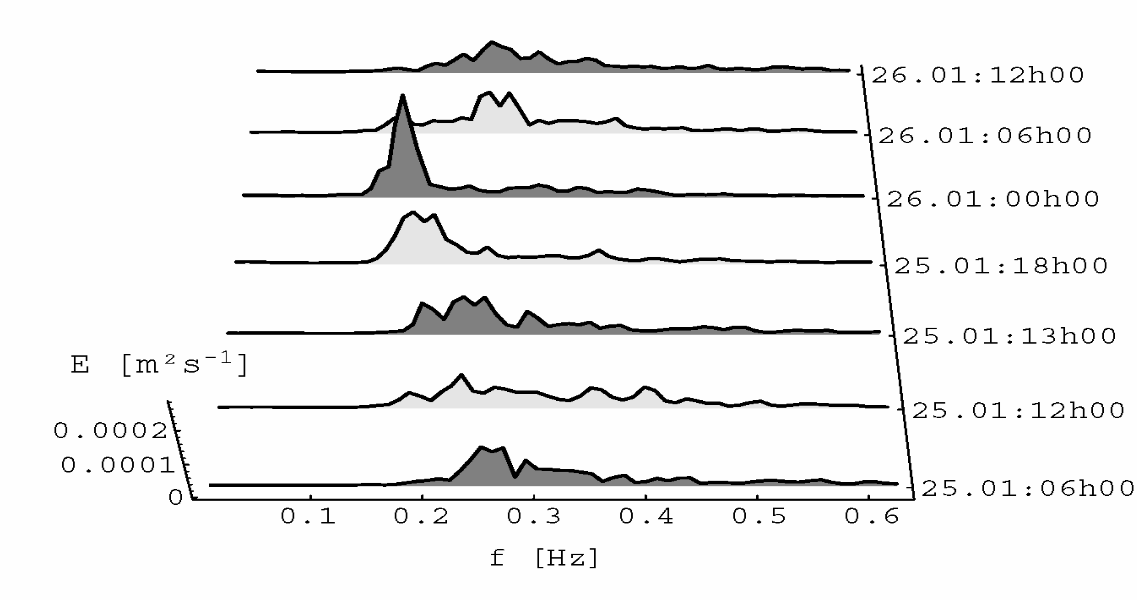


Fig. 2 Dimensionless frequency spectra; waves measured to a depth of 3.5 m, 100 m from the coastline.

3.5 m (mean water level). The wave frequency of the significant waves moves from a typical value for the Baltic of 0.25 Hz to 0.16 Hz and quickly increases to the same value of 0.25 Hz (see Figure 2). Due to these unusual conditions the waves act only for 3 hours on the foreshore and this region was not wash over for many days after the storm and the topography was defended for erosion.

## Methodology

After the storm, asymmetric beach cusps were formed with a quasi-uniform longshore wavelength of 8 m, which was measured over 10 cusps from cusps top to cusps top. We studied the topography in detail in a 10 m x 9 m section normal to the coast. The

topography in the dry fallen foreshore cups region was measured on a raster of 1 m alongshore and 0.5 m cross-shore over one beach-cusp wavelength. This measured and for numerical study extrapolated topography is shown in Figure 3.
The basic data set for this study are measurements of 1) waves at a depth of 3.5 m for 7 minutes every 6 hours; 2) water level every hour; and 3) continuous wind measurements. The sampling interval for waves is 0.2 sec with a vertical resolution of 3 cm. Power spectra were calculated from the wave data and are shown in Figure 3. The wave spectra during high water were put in the non-linear wave model BOWAM2 of the University of Hanover (Schröter, 1991). The measured topography was interpolated on a grid of 0.5 m x 0.5 m and then put into the model. The model has 96 cross-shore and 92 longshore grid points. The boundary conditions are open on the sea side of the model and vary from closed (sea side) to open (shoreline) on the cross-shore boundaries. These boundary conditions give the possibility for edge-wave excitation of up to 40 meters in wavelength in the two longshore directions. We have the results from two runs 1) with the measured topography and 2) similar to 1) but with an average of the topography over all grid points alongshore. To obtain the edge-wave energy, a Fourier longshore wavelength spectrum was employed, where the infinite wavelength ($\lambda \rightarrow \infty$) represents the incoming wave.

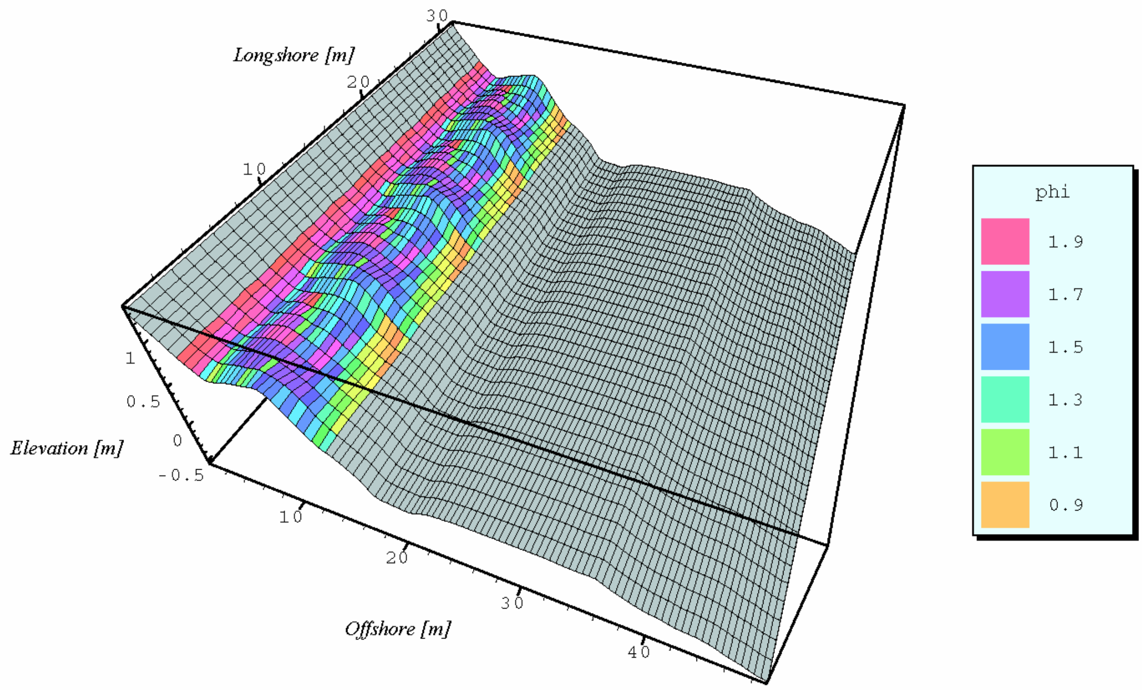


Fig. 3 The topography examined by the numerical model BOWAM2. Shown also in the picture is the grain size of an area of 8x12 m in phi=$\log_2$(mm). This area is duplicated. The grain-size results are not discussed in the text.

Let $x$ and $y$ be distance from the shore and distance alongshore, respectively. The time is denoted by $t$. We examined numerically the edge-wave dispersion relation and the edge-wave amplitude on the topography, which is the same as in numerical run 2), that is variable in the cross-shore direction $x$ but constant in the longshore direction $y$, by assuming that the cross-shore velocity $u$, the longshore velocity $v$, and the sea-surface elevation $\eta$ are wave-like in the longshore direction

$$\eta(\mathrm{x},\mathrm{y},\mathrm{t}) = \eta^*(x)e^{i(ky-\sigma t)}, \qquad \textbf{(1)}$$

where $k$ and $\sigma$ are the longshore radian wave number and wave frequency, respectively and $\eta^*(x)$ the edge wave amplitude on a barred topography $h(x)$. For a flat beach $\eta^*(x) \sim L_n(x)$ where $L_n(x)$ denote the Laguerre Polynomals. Substituting (1) and analogous equations for $u$ and $v$ into the irrotational, inviscid shallow water equations of momentum and continuity gives

$$\left[\frac{gh\eta_x^*}{\sigma^2}\right]_x + \eta^*\left(1-\frac{k^2gh}{\sigma^2}\right) = 0, \qquad \textbf{(2)}$$

the classic edge-wave differential equation, where $h$ is the water depth, $g$ is the gravitational acceleration, and subscripts indicate differentiation. The differential equation (2) can be solved numerically by a Runge-Kutta algorithm starting with the boundary conditions $hu$=0 at $x$=0 and the requirement $\eta \rightarrow 0$ for $x \rightarrow \infty$ (Holman and Bowen, 1979). For a given frequency $\sigma$, there is a set of edge waves of different modal numbers $n$ (where $n$=0, 1, 2, ...) with the longshore radian wave number $k_n$ .

## Results

The topography of the beach after a storm is shown in Figure 3 and 4b. Periodic erosions (and/or accretional processes) are seen in the swash zone after the storm. The topography has a quasi-uniform longshore wavelength of 8 m. The beach was uniform in slope from the dune to the swash zone before the storm of January 25th – 26th 1993. With the measured topography we examined the dispersion relation on the edge wave wavelength of 8 m. First we used Eq. (2) and the measured topography averaged in longshore direction as described above. The mode-zero edge wave frequency belonging to the wavelength $\lambda_0$ = 8 m is 0.16 Hz. That is the same frequency as the measured significant incident wave frequency on high water. These incident waves acted only for 3 hours on the foreshore.

Equation (2) gives a solution for constant beach profile alongshore. Schönfeldt (1995) showed that edge waves can be trapped on bars and longshore currents. On bars and currents trapped edge waves have here the maximum of amplitude contrary to edge waves on planar beaches with uniform slope. Therefore we tested the influence of the alongshore not uniform cusps profile on the edge waves in a non-linear two-dimensional wave model BOWAM2. On the sea side we used the wave spectra during high water as boundary conditions and the two-dimensional sea-bed topography in the two cases: 1) the topography measured after storm plus high water level and 2) similar to 1) but with averaging in longshore direction. After Fourier transformation of the calculated time series in space we get the longshore wavelength spectrum. We calculated the edge-wave amplitude for the wavelength at $\lambda$ = 8 m in the two cases.

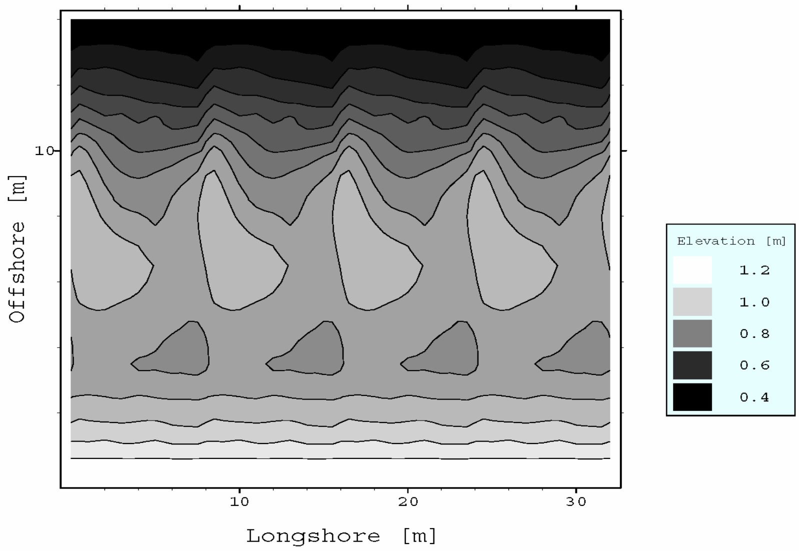


Fig. 4a Measured topography. Since high resolution topography data fail for understanding we have it duplicated alongshore.

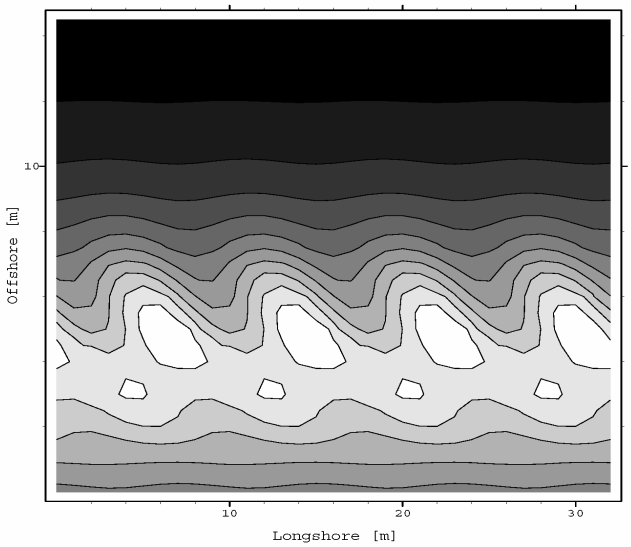


Fig. 4b The norm of the superimposition velocity of the incoming wave with an amplitude of 1.0 and the synchronous mode-zero edge wave running from west to east (to the right when one look to the sea) with an amplitude of 0.2 with constant phase-coupling between this waves.

The ratio of amplitude $a_1/a_2$ and the edge wave frequency are present in Figure 5. This ratio of amplitude is an average in cross-shore direction of all spectral elements with a wavelength of $\lambda$ = 8 m, but the highest amplitudes were found 5 m away from the coast at the top of the cusps during high water. In agreement with Eq. (2) this is the region of mode-zero edge waves. In the model run with the measured two-dimensional sea-bed topography, the amplitudes were up to 18 times greater than in the case 2). We have a slight asymmetry in the amplitude ratio. The waves running from the left to the right are something preferentially forced. This is an influence of the topography only. Note, the incident waves running normal towards the coast in the model. The higher edge-wave modes are also amplified but not so much. Another influence of the sea-bed topography is an asymmetry in edge wave frequency, edge waves running from the right to left are less intense forced and have a lower frequency. Equation (2) and the model runs do not give the same edge-wave dispersion relation (see Fig. 5), Equation (2) gives the free wave solution and the model gives the equilibrium solution forced by wave braking and damped down by friction.

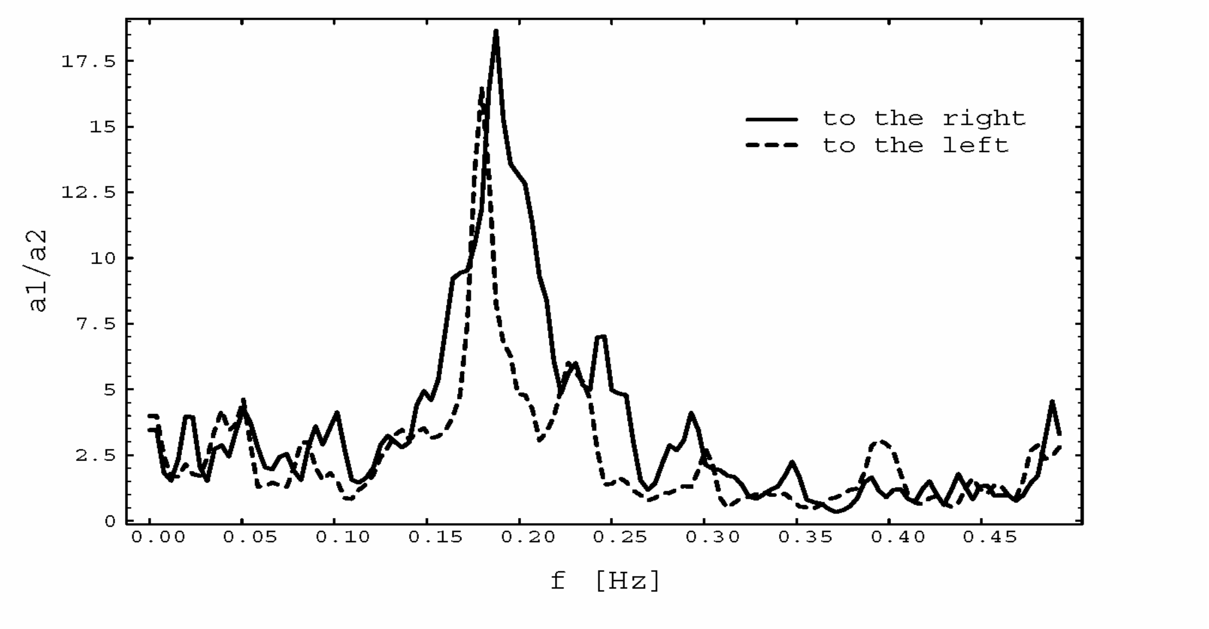


Fig. 5 The ratio of the edge-wave elevation $a_1$ with an 8 m wavelength on a rhythmic alongshore sea-bottom topography (Fig. 4a) to the edge-wave elevation $a_2$ on the same sea-bed profile but with a mean over all longshore grid points.

The directions denote the propagation of the edge waves when you look to the sea. The elevations were determined by the numerical model BOWAM2.

The incident wave amplitude can be calculate analogous to the edge wave by Eq. (2) with $k = 0$ (wave number in lonshore direction). This calculation gives the amplitudes for a standing wave on variable topography in cross-shore direction. As a result, the amplitudes for an incoming wave can be reconstructed as envelope of the standing wave amplitude (Bessel function on uniform slope).

The cross-shore velocity $u$ and the longshore velocity $v$ of the synchronous mode-zero edge wave and the incoming wave are calculated by the shallow water equations of momentum and continuity with the edge wave amplitude $\eta^*$ calculated by Eq. (2). The two components $u$ and $v$ of superimposition of an incoming wave and the synchronous mode-zero edge wave running from west to east (from the left to the right when looking seaward) was calculated with constant-phase coupling between these waves and without wave breaking. In principle these restrictions do not disturb the result and conclusions. The norm of resulting velocity $u$ and $v$ of a superimposition of incoming wave with the arbitrary amplitude of 1.0 and the synchronous mode-zero edge wave with amplitude of 0.2 of the incident wave is shown in Figure 4b. This picture can only exist if phase-coupling between the waves occurs. At the occurrence of every incident wave crest, synchronous edge waves will cause a longshore spacing of wave-height maximums equal to one synchronous edge-wave wavelength (Bowen and Inman, 1969, Sallenger, 1979, Schwarzer, 1989). Evidence for phase-coupling between edge-wave modes is given by Huntley (1988). Figure 4a and 4b look similar.

## Discussion

„If a beach were exposed to waves having an exactly symmetrical, orbital velocity, all the sediment would slide down the slope and out to sea. The existence of the beach depends on small departures from symmetry in the velocity field balancing this tendency for gravity to move material offshore“ (Bowen, 1980). Alongshore departure from symmetry in the velocity field will be produced by the superimposition of incident wave and mode zero edge wave (Fig.4b). This superimposition has the structure of a standing wave where the nodes of standing wave are replaced by minima of the drift velocity. Suspended load tends to accumulate at these minima. We have a positive feedback from the morphology to initial edge waves. The periodic crests and troughs in a line at a distance of 5m from the coast force waves are formed during the wave run-up. The mode-zero edge wave has the greatest excitation probability.

The initial topography has no structure. At the beginning the mode-zero edge wave are excited without resonance, the amplitudes will be small. If we have in one place a random structure that agrees with the edge-wave wavelength, then phase-coupling and the positive feedback begin. The periodicity in a wave-induced current (incident wave and edge wave) causes a periodicity in sediment transport. The resulting topography has a positive feedback on edge waves, and so on. This initial event will spread over the entire coastline. The initial process can begin on several locations at the same time. To the extent that the sea-bed topography changes from alongshore constant to rhythmic, the energy of edge waves causes the crests and troughs to rise.

Standing sub harmonic edge waves can also cause a rhythmic topography. The sub harmonic edge wave with the peak frequency of the wave spectrum has a wave length of 16 m, and the superimposition of two waves running in opposite directions extinguish and reinforce themselves with a spacing of 8 m. But the resulting drift velocity of superimposition is strongly symmetrical and not asymmetrical like the measured topography (cf. Fig. 4a).

Huntley and Bowen (1975) observed shoreline beach cusps and noted an apparent correspondence between the spacing of cusps found in the field and that due to n = 0 synchronous edge waves. Figure 4a (measured topography) and 4b (superimposed drift velocity) are similar but not mirror-image exact. Lines of equal phase of the superimposed wave are not perpendicular to the shore.

It has been shown both theoretically (Bowen and Guza, 1978) and in field data (Oltman-Shay and Guza, 1987; Oltman-Shay et al.,1989; Bryan and Bowen, 1998) that, under obliquely incident wave conditions, edge waves may be preferentially forced in the same direction as the forcing of longshore current. The incoming waves (wave direction NW in deep water) will preferentially force edge waves in the direction from west to east. Note that the numerical model also preferentially forces edge waves running to the right (Fig. 5) though we have deliberately chosen a wave direction perpendicular to the coast. We have two reasons for excitation of synchronous edge waves running from west to east (left to right): the wave direction NW (coast line west east the sea in north direction) and the preferentially forcing of edge waves by the cusps profile even if the wave run up is perpendicular to the coast.

## Conclusions

There are two theories with any real credibility as to why beach cusps are formed, the standing edge wave theory and the self-organisation theory. The latter discuss that positive feedback between the morphology of the beach and the flow of the water creates relief patterns. The problem of this theory is that this method of cusp formation would take time and if one observes their formation, then one would see a number of random cusps form along the beach, which then slowly spread along the shore as they even out in size, with small cusps joining together and larger cusps being separated in two. But in the field, cusps form a regular pattern almost instantly and they all appear at the same time.

Our results suppose that for unsymmetrical beach cusps (periodic but not mirror-image exact) both of the two theories must be taken into account. One or more of the number of random cusps will be suitable for resonant excitation of edge wave and phase-coupling between edge-wave and incident wave. In this manner the here starting edge wave affect the neighbouring random cusps and bring these in “phase”. The missing link in the self-organisation theory are the edge waves. We can not say that this causal chain act in any case of beach cusps generation. We only have this hints for asymmetric periodic but not mirror-image exact beach cusps.

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## References

Almar, R., Coco, G., Bryan, K.R., Huntley, D.A., Short, A.D. and Senechal, N., 2008: Video observations of beach cusp morphodynamics. Marine Geology, 254, 216-223

Bryan, K. R. and Bowen, A. J., 1996: Edge waves trapping and amplification on barred beaches. J. Geophys. Res. 101 (C3), 6543-6552.

Bryan, K. R. and Bowen, A. J., 1998: Bar-trapped edge waves and longshore currents. J. Geophys. Res. 103 (C12), 27867-27884.

Bowen, A. J. and Inman, D. L., 1969: Rip currents, 2. Laboratory and field observations. J. Geophys. Res. 74, 5479-5490.

Bowen, A. J. and Guza, R. T., 1978: Edge waves and surf beat. J. Geophys. Res. 83, 1913-1920.

Bowen, A.J., 1980. Simple models of nearshore sedimentation; beach profiles and longshore bars. In: McCann S.B. (Ed.), The Coastline of Canada, Geological Survey of Canada, Paper 80-10: 1-11.

Coco, G., T.J. O'Hare and Huntley, D.A., 1999: Beach cusps: a comparison of data and theories for their formation, J. Coast. Res. 15 (3), pp. 741-749.

Dolan, R., L. Vincent, and Hayden, B., 1974: Crescentic coastal landforms. Z. Geomorph. 18, 1-12.

Fredsøe, J. and Deigaard, R., 1992: Mechanics of coastal sediment transport. Advanced Series on Ocean Engineering, Volume 3.

Guza, R. T., and Inman, D. L., 1975: Edge waves and beach cusps. J. of Geopy. Res. 80, 2997-3012.
Holman, R. A. and Bowen, A. J., 1979: Edge waves on complex beach profiles. J. Geophys. Res. 84, 6339-6346.
Holman, R. A. and Bowen, A. J., 1982: Bars, bumps, holes: Models for the generation of complex beach topography. J. Geophys. Res. 87, 457-468.
Hom-ma, M., and Sonu, C.,1963: Rhythmic patterns of longshore bars related to sediment characteristics, In: Proceedings 8th Conference on Coastal Engineering, American Society of Civil Engineering, New York, pp. 248-278.
Howd, P.A., Bowen, R. A., Holman, R. A., 1992: Edge waves in strong longshore currents. J. Geophys. Res. 97**,** 11357-11371.
Huntley, D. A. and Bowen, A. J., 1975: Field observations of edge waves and their effect on beach material. J. Geol. Soc. Lond. 131, 69-81.
Huntley, D. A., 1988. Evidence for phase coupling between edge wave modes. J. Geophys. Res. 93, 12392-12408.
Kirby, J. T., R. A. Dalrymple, and P. L. F. Lui, 1981: Modification of edge waves by barred-beach topography, Coastal Eng. 5, 35-49.
Komar, P. D., 1981: Rhythmic shoreline features and their origins, In: Gardener, R., et al. (Ed.), Large-scale Geomorphology, Oxford University Press, New York.
Miller, J.R., Orbock Miller, S.M., Torzynski, C.A. and Kochel, R.C., 1989: Beach-cusp destruction, formation, and evolution during and subsequent to an extratropical storm. Duck, North Carolina. J. of Geology 97, 749-760.
Oltman-Shay, J. and Guza, R.T., 1987: Infragravity edge wave observations on two California beaches. J. Phys. Oceanography 17, 644-663.
Oltman-Shay, J., Elgar, S., Howd, P., 1989: Observations of infragravity-frequency long waves, II. Comparisons with a 2-D wave group generation model (abstract). Eos Trans. AGU 70, 1333.
Sallenger Jr., A.H., 1979: Beach-cusp formation. Mar. Geol. 29, 23-37.
Schönfeldt, H.-J., 1989: Are edge waves responsible for the location of sand reefs ? Beitr. zur Meereskunde 60, 35-40.
Schönfeldt, H.-J., 1991: Randwellen und Sedimentation. Geophysikalische Veröff. der Universität Leipzig 4, 75-87.
Schönfeldt, H.-J., 1991. Dispersionsbeziehungen von Randwellen auf natürlichen Bodenprofilen mit uferparallelen Sandbänken. Beitr. Meereskd. 62, 53-68.
Schönfeldt, H.-J., 1994: Randwellen in der Ostsee und anormale Dispersion in der Brandungszone. D. Hydrogr. Z. 46, 81-98.
Schönfeldt, H.-J., 1995: On the modification of edge waves by longshore currents. Continental Shelf Research 15**,** 1213-1220.
Schröter, A., 1991. Das numerische Seegangsmodell BOWAM2 1990 - Grundlagen und Verifikationen. Univ. Hannover, Inst. f. Strömungsmechanik, Bericht 31, Teil 2.
Schwarzer, K., 1989: Sedimentdynamik in Sandriffsystemen einer tidenfreien Küste unter Berücksichtigung von Rippströmen. Berichte-Reports, Geol.- Paläont. Inst. Univ. Kiel, 33.
Werner, B.T. and Fink, T.M., 1993: Beach cusps as self-organised patterns, Science 260, pp. 968-971.